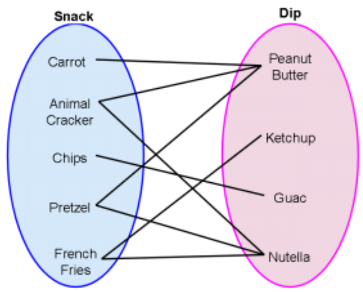
Relations and Functions

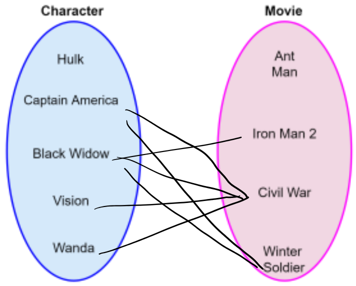
Unit 1 Day 1: 1.1-1.4

Today
- Function Vocab
- Connecting Verbal Descriptions and Graphical Representations of Functions

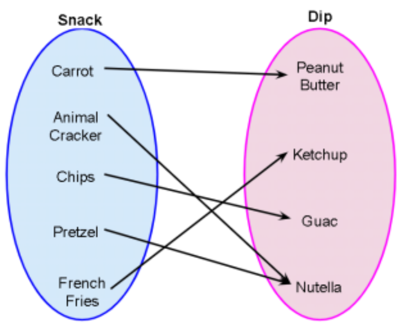
Homework: Function Vocab and Rates of Change Worksheet (on Canvas)

Relation: Any rule of correspondence between the elements of two sets





Function: A relation that maps each element in one set (called the domain or input values) to exactly one element in another set (called the range or output values)



Sometimes the elements of the domain are called the "independent variable" or "input values." We frequently call them x-values.

Sometimes the elements of the range are called the "dependent variable" or "output values." We frequently call them y-values.

Do these make sense? Read/Listen, ask if there's a question raised in your mind, then write anything you think you need to note. But if you ask me to pause before going to the next slide, and I see you writing this down word for word, I will throw a duck at you. Don't worry, the duck will be fine. They can fly.

Characteristics of a Function from set A to set B

- Each element of A must be matched with an element of B
- Some elements of B may not be matched with any element of A
- Two or more elements of A may be matched with the same element of B
- An element of A (the domain) cannot be matched with two different elements of B

Ways We Communicate Functions

Table

0	1
6	1.40
12	5.40
18	7.50
24	8.50
30	6.40

Verbal Description

The US Inflation rate decreased over the last eleven months of the study. The inflation rate seemed to have a point of inflection in December of 2020. The maximum value of the inflation rate was 9.1%.

Graph

Equation

$$R(t) = 0.698t^4 - 4.656t^3 + 15.797t^2 - 23.223t + 11.535$$

Zeros of a Function

Definition: an input-value that corresponds with an output-value of zero.

In a Table or List of Points

Look for the output-value of zero. The corresponding input-value is a zero.

x	$f(x)$
-10	2
-5	0
0	-4
5	-3
10	11
15	0

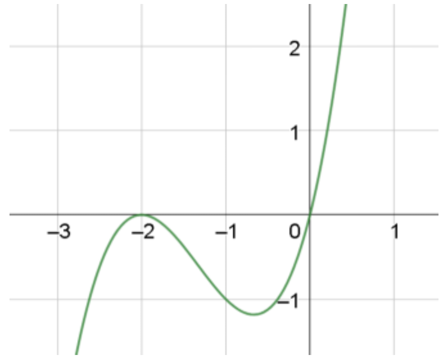
With an Equation

Plug in 0 for the output (often y or $f(x)$). Solve for the input (often x or t). The solutions are the zeros.

$$g(t) = t^2 - 9$$

With a Graph

Look at the x-intercepts. The zeros are the x-values of these points.



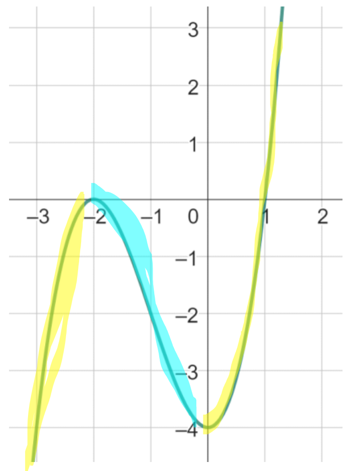
Increasing/Decreasing

A function f is increasing on an interval when, for any two numbers a and b in the interval, $a < b$ implies $f(a) < f(b)$

In simpler terms, f is increasing over an interval if no matter which two numbers you pick in the interval, the bigger x-value corresponds with the bigger y-value.

A function f is decreasing on an interval when, for any two numbers a and b in the interval, $a < b$ implies $f(a) > f(b)$

In simpler terms, f is decreasing over an interval if no matter which two numbers you pick in the interval, the bigger x-value corresponds with the smaller y-value.



inc: $(-\infty, -2]$

dec: $[-2, 0]$

inc: $[0, \infty)$

We describe where a function is increasing/decreasing by giving an interval of x-values. NOT y-VALUES

Increasing/Decreasing

With a Table

You cannot tell for sure that a function is increasing or decreasing over an interval, but you can tell for sure if it isn't increasing or decreasing.

x	-3	-1	0	2	5	8	9	10
$f(x)$	1	0	5	8	10	14	11	19

With an Equation

Given only the equation of a function, sometimes we can apply our understanding of transformation of functions to know where that function will increase vs. decrease, but for the most part doing increasing vs. decreasing with just an equation is a calculus topic, not pre-calculus.

Absolute Extrema

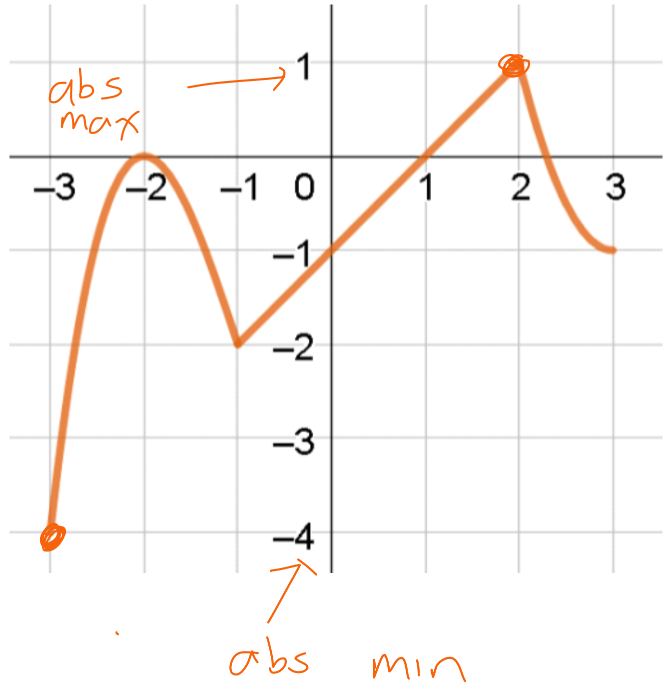
The max is f (2) = 1

Absolute Maximum: The highest output-value of a function

Absolute Minimum: The lowest output-value of a function

Some nomenclature:

- The plural of maximum is maximums or maxima
- The plural of minimum is minimums or minima
- The word that groups maxima and minimum together is "extrema." Connect this in your mind to maxima and minima being the extremes of the graph.
- Sometimes we omit the word "absolute."
- Sometimes we use the word "global" instead of "absolute"
- "Find the maximum" = "Find the absolute maximum" = "Find the global maximum"



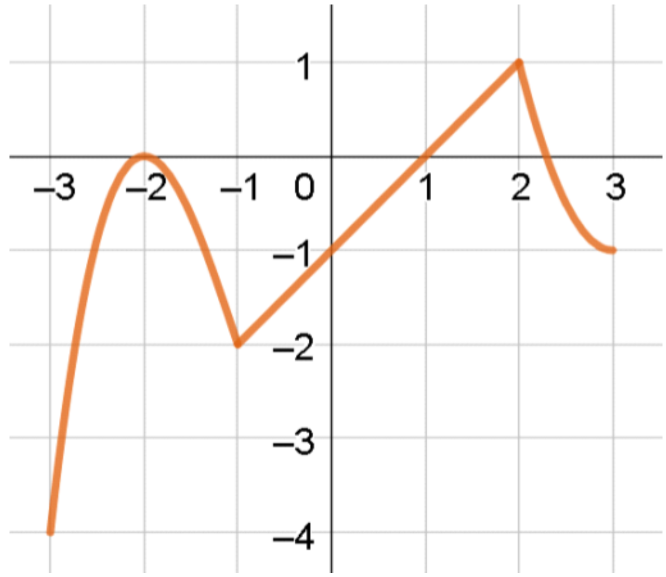
Weird Question…

Why bother having a plural for the word maximum or minimum? Can't only one y-value be higher than all the other y-values and only one y-value be lower than all the other y-values? So then why would we need a word for if there is more than one maximum?

Relative Extrema

Relative Maximum: An output-value higher than the output-values immediately neighboring it

Relative Minimum: An output-value lower than the output-values immediately neighboring it



Some things:

- We can still use the word "extrema" to group these together. A question may say "Find all relative extrema"
- Sometimes we use the word "local" instead of "relative"
- A function can have many relative extrema, but it can only have at most one absolute max and at most one absolute min.
- Remember, if the question just says "maximum" or "minimum," it means absolute, not relative.

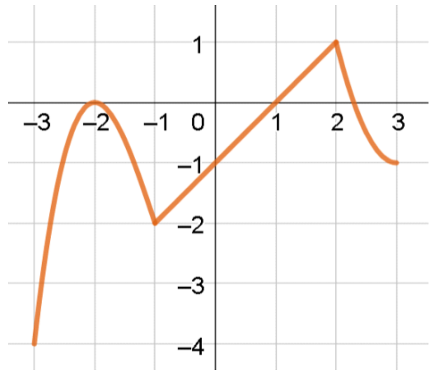
HOW TO KNOW IF A QUESTION IS TALKING ABOUT THE x-VALUE OR y-VALUE OF AN EXTREMUM

$f(2)=1$ is an abs max

When asked "where" does $f(x)$ have extrema, answer with an x-value.

When asked for the extrema of $f(x)$ answer with a y-value.

If you aren't sure, you can play it safe and tell me "The absolute maximum is $f(2) = 1$." This way, you covered all your bases and to me it just looks like you were showing your work. If you give it as the point $(2,1)$ I won't be as forgiving.



Example 1

List all zeros of $f(x)$

$X = -3$ $\quad$ $X = -1$

Over what interval or intervals does $f(x)$ increase?

$(-2, 2)$ and $(3, 5)$

Find the absolute maximum and minimum values of $f(x)$

4 $\quad$ -1

Where does $f(x)$ have relative maxima?

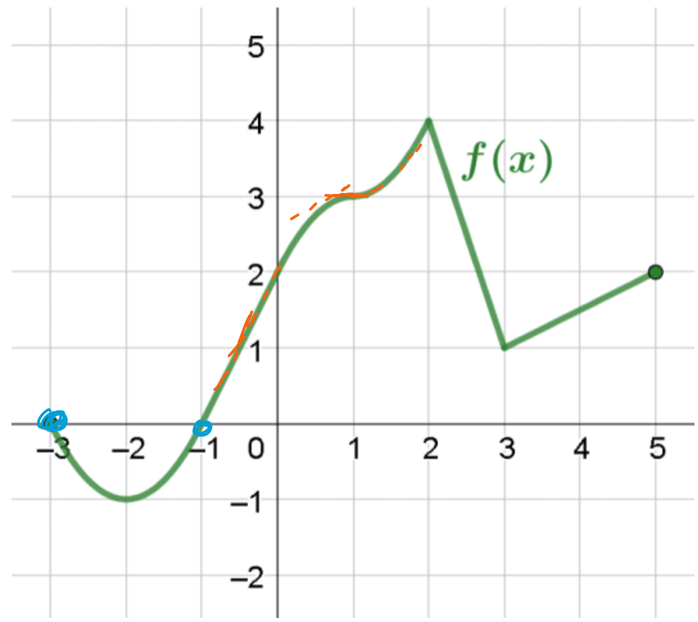
$X = -3, 2, 5$

What is the domain of $f(x)$?

$[-3, 5]$

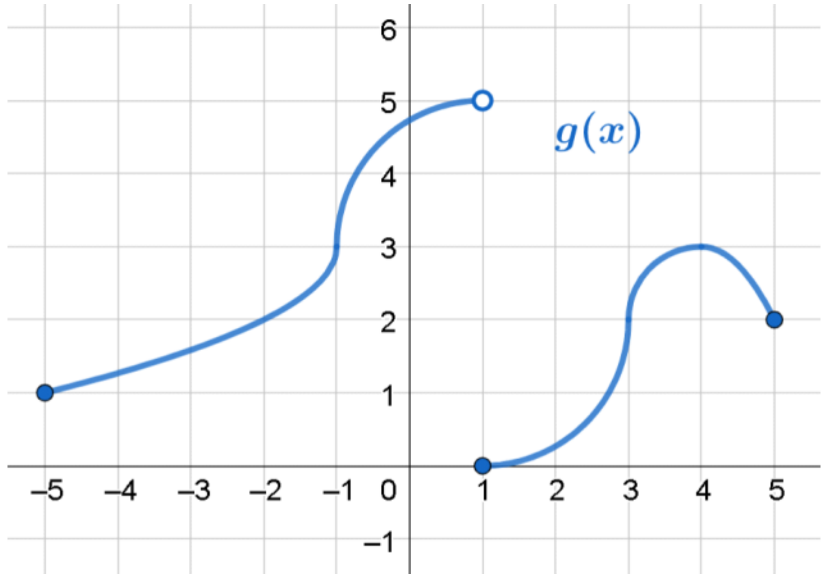
What is the range of $f(x)$?

$[-1, 4]$



Example 2

Find all zeros of $g(x)$



Find all intervals over which $g(x)$ increases and all intervals over which $g(x)$ decreases

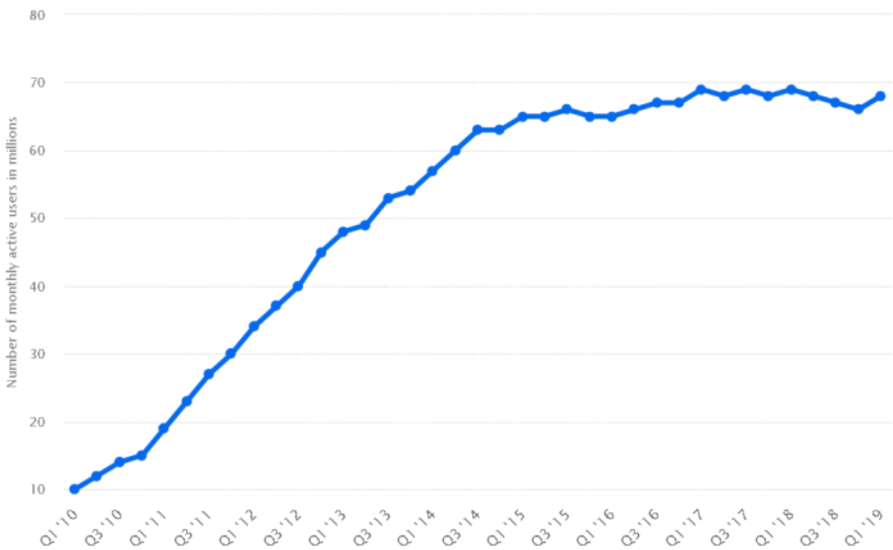
At what x-values do the absolute extrema of $g(x)$ occur?

Find all local maxima of $g(x)$

Rate of Change of a Function: How quickly the output-value of a function changes per change in the input-value.

Examples:

- The volume of the balloon is increasing by five cubic inches per second
- The water level in the reservoir is decreasing by one foot per week
- The slope of the graph is $\frac{3}{2}$.



To the left is a graph of the number of active twitter users as a function of time.

A normal person asks: "When was twitter growing the fastest?"

An mathematician asks: "When was the rate of change of the number of twitter users greatest?"

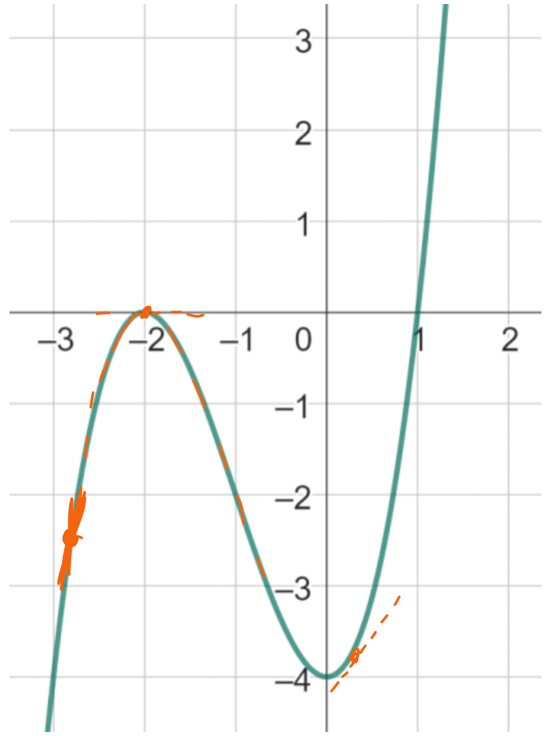
Two Types of Rate of Change

Average Rate of Change

- Questions about this always give an interval
- Formula for average rate of change over the interval $[a, b]$: $\frac{f(b)-f(a)}{b-a}$

Instantaneous Rate of Change

- You won't have to actually calculate it. Just comment on it and maybe estimate it
- We won't usually use the word "instantaneous" to save time, but you can use it to add clarity.



The term "Rate of Change" should be intimately connected to the word "slope" in your head. If your phone always autocorrected "rate of change" to "slope" it will not interfere with your life.

Two Types of Rate of Change
Example Questions

$$\frac{f(1) - f(-3)}{(1) - (-3)} = \frac{0 - -4}{1 - -3}$$

$$\frac{4}{4}$$

$$1$$

Find the average rate of change of $f(x)$ over the interval $[-2,0]$

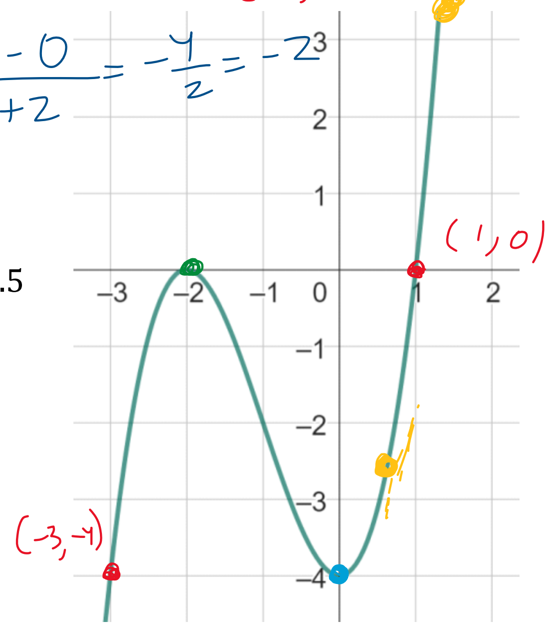
$$\frac{f(0) - f(-2)}{(0) - (-2)} = \frac{-4 - 0}{0 + 2} = -\frac{4}{2} = -2$$

Find the average rate of change of $f(x)$ over the interval $-3 \leq x \leq 1$

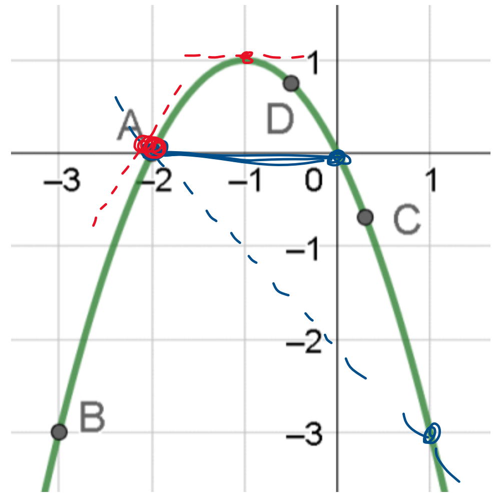
Which is greater, the rate of change of $f(x)$ at $x = 0.5$ or the rate of change of $f(x)$ at $x = 1.5$?

Over what interval(s) is the rate of change of $f(x)$ negative?

(-2, 0)



Example 3 - Rate of Change and Graphs



Find the average rate of change of $g(x)$ over the interval $[-2,1]$

$$\frac{g(1) - g(-2)}{(1) - (-2)} = \frac{-3 - 0}{1 - -2} = \frac{-3}{3} = -1$$

Which labelled point has the greatest rate of change? Which labelled point has the least rate of change?

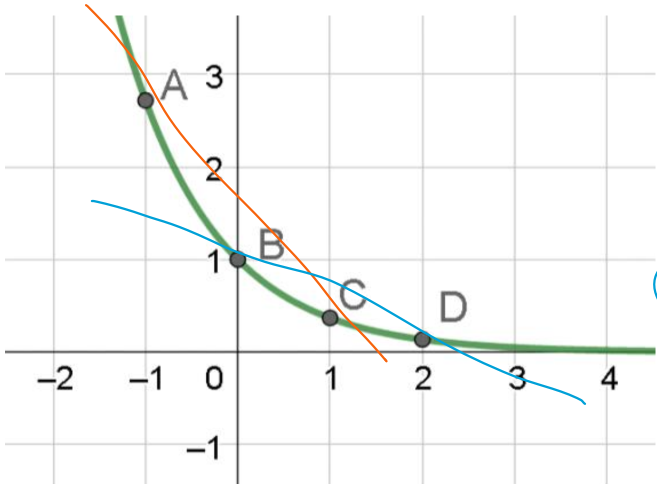
C   B

Give an example of an interval over which the average rate of change of $g(x)$ is zero.   [-2, 0]
[-3, 1]

Which of these is a reasonable guess for the rate of change at the point labelled A?

A. ~~-4~~   B. ~~0~~   C. $\frac{3}{2}$ D. 8

Example 4a



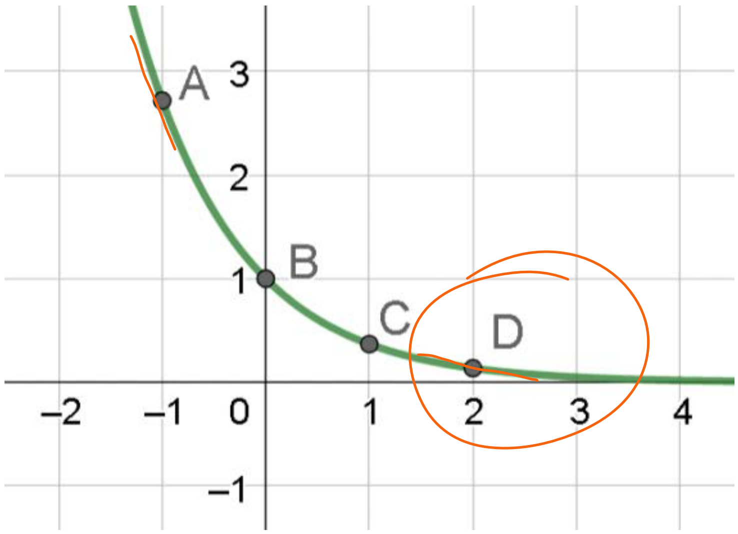
Which is Greater?

The average rate of change of $f(x)$ from point A to point C

Vs.

The average rate of change of $f(x)$ from point B to point D

Example 4b



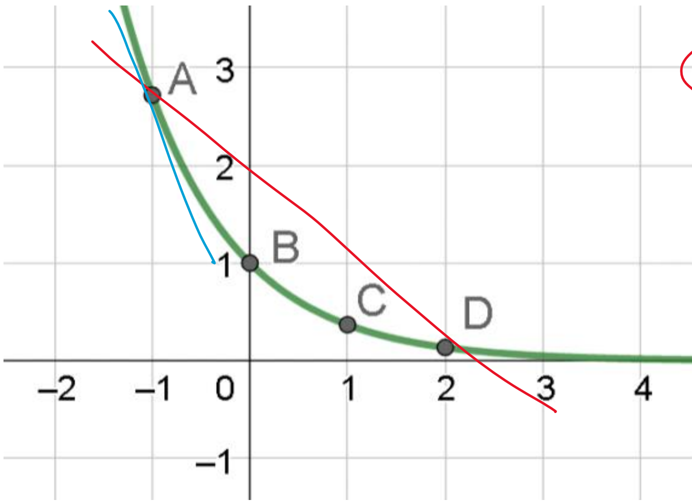
Which is Greater?

The rate of change of $f(x)$ at point A

Vs.

The rate of change of $f(x)$ at point D

Example 4c



Which is Greater?

The average rate of change of $f(x)$ from point A to point D

Vs.

The rate of change of $f(x)$ at point A

Some points not to be overlooked. Don't trust me on these. Think to yourself "does that make sense" and then ask me if it doesn't.

- If the rate of change of $f(x)$ is positive on the interval $[a, b]$, then $f(x)$ is increasing on that interval.

- If the rate of change of $f(x)$ is negative on the interval $[a, b]$, then $f(x)$ is decreasing on that interval.

- If the rate of change of $f(x)$ is constant everywhere on the interval $[a, b]$, then over that interval $f(x)$ is linear.

- If the rate of change of $f(x)$ is 0 everywhere on the interval $[a, b]$, then over that interval $f(x)$ is constant